Symbolab

Solution

Extreme Points of $\sin(x)-\cos(x), 0\le x\le 2\pi$: Maximum$\left(\frac{3\pi}{4}, \sqrt{2}\right)$, Minimum$\left(\frac{7\pi}{4}, -\sqrt{2}\right)$

Steps

First Derivative Test definition

Suppose that $x=c$ is a critical point of $f(x)$ then,
If $f'(x)>0$ to the left of $x=c$ and $f'(x)<0$ to the right of $x=c$ then $x=c$ is a local maximum.
If $f'(x)<0$ to the left of $x=c$ and $f'(x)>0$ to the right of $x=c$ then $x=c$ is a local minimum.
If $f'(x)$ is the same sign on both sides of $x=c$ then $x=c$ is neither a local maximum nor a local minimum.

Find the critical points: $x=\frac{3\pi}{4}, x=\frac{7\pi}{4}$ *Hide Steps*

Critical point definition

Critical points are points where the function is defined and its derivative is zero or undefined

Find where $f'(x)$ is equal to zero or undefined

$f'(x)=\cos(x)+\sin(x)$ *Hide Steps*

$\frac{d}{dx}(\sin(x)-\cos(x))$

Apply the Sum/Difference Rule: $(f\pm g)'=f'\pm g'$

$=\frac{d}{dx}(\sin(x))-\frac{d}{dx}(\cos(x))$

$\frac{d}{dx}(\sin(x))=\cos(x)$ *Hide Steps*

$\frac{d}{dx}(\sin(x))$

Apply the common derivative: $\frac{d}{dx}(\sin(x))=\cos(x)$

$=\cos(x)$

$\frac{d}{dx}(\cos(x))=-\sin(x)$ *Hide Steps*

$\frac{d}{dx}(\cos(x))$

Apply the common derivative: $\frac{d}{dx}(\cos(x))=-\sin(x)$

$=-\sin(x)$

$=\cos(x)-(-\sin(x))$

Factor

$=\cos(x)+\sin(x)$

Solve $\cos(x)+\sin(x)=0$: $x=\frac{3\pi}{4}, x=\frac{7\pi}{4}$ *Hide Steps*

$\cos(x)+\sin(x)=0, 0\le x\le 2\pi$

Divide both sides by $\cos(x)$, $\cos(x)\neq 0$

$$\frac{\cos(x)+\sin(x)}{\cos(x)}=\frac{0}{\cos(x)}$$

Refine

$$\frac{\sin(x)}{\cos(x)}+1=0$$

Use the following identity: $\frac{\sin(x)}{\cos(x)} = \tan(x)$

$\tan(x) + 1 = 0$

Subtract 1 from both sides

$\tan(x) + 1 - 1 = 0 - 1$

Simplify

$\tan(x) = -1$

General solutions for $\tan(x) = -1$

$x = \frac{3\pi}{4} + \pi n$

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Solutions for the range $0 \le x \le 2\pi$

$x = \frac{3\pi}{4}, x = \frac{7\pi}{4}$

$x = \frac{3\pi}{4}, x = \frac{7\pi}{4}$

Identify critical points not in $f(x)$ domain

Domain of $\sin(x) - \cos(x), 0 \le x \le 2\pi: \quad 0 \le x \le 2\pi$ *Hide Steps*

Domain definition

The domain of a function is the set of input or argument values for which the function is real and defined

Set the domain to the given interval

$0 \le x \le 2\pi$

The function has no undefined points nor domain constraints. Therefore, the domain is

$0 \le x \le 2\pi$

All critical points are in domain

$x = \frac{3\pi}{4}, x = \frac{7\pi}{4}$

Domain of $\sin(x) - \cos(x), 0 \le x \le 2\pi: \quad 0 \le x \le 2\pi$ *Hide Steps*

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Combine the critical point(s): $x = \frac{3\pi}{4}, x = \frac{7\pi}{4}$ with the domain

The function monotone intervals are:

$0 \le x < \frac{3\pi}{4}, \frac{3\pi}{4} < x < \frac{7\pi}{4}, \frac{7\pi}{4} < x \le 2\pi$

Check the sign of $f'(x) = \cos(x) + \sin(x)$ at each monotone interval

Check the sign of $\cos(x) + \sin(x)$ at $0 \le x < \frac{3\pi}{4}$: Positive *Hide Steps*

Evaluate the derivative at a point on the interval. Take the point $x = 1$ and plug it into $\cos(x) + \sin(x)$

$\cos(1) + \sin(1)$

Simplify

$\sin(1)+\cos(1)$

Refine to a decimal form

$1.38177\ldots$

Positive

Check the sign of $\cos(x)+\sin(x)$ at $\frac{3\pi}{4}<x<\frac{7\pi}{4}$: Negative

Hide Steps

Evaluate the derivative at a point on the interval. Take the point $x=4$ and plug it into $\cos(x)+\sin(x)$

$\cos(4)+\sin(4)$

Simplify

$\sin(4)+\cos(4)$

Refine to a decimal form

$-1.41045\ldots$

Negative

Check the sign of $\cos(x)+\sin(x)$ at $\frac{7\pi}{4}<x\le 2\pi$: Positive

Hide Steps

Evaluate the derivative at a point on the interval. Take the point $x=6$ and plug it into $\cos(x)+\sin(x)$

$\cos(6)+\sin(6)$

Simplify

$\sin(6)+\cos(6)$

Refine to a decimal form

$0.68075\ldots$

Positive

Summary of the monotone intervals behavior

	$0\le x<\frac{3\pi}{4}$	$x=\frac{3\pi}{4}$	$\frac{3\pi}{4}<x<\frac{7\pi}{4}$	$x=\frac{7\pi}{4}$	$\frac{7\pi}{4}<x\le 2\pi$
Sign	$+$	0	$-$	0	$+$
Behavior	Increasing	Maximum	Decreasing	Minimum	Increasing

Plug the extreme point $x=\frac{3\pi}{4}$ into $\sin(x)-\cos(x) \quad\Rightarrow\quad y=\sqrt{2}$

Maximum$\left(\frac{3\pi}{4},\sqrt{2}\right)$

Plug the extreme point $x=\frac{7\pi}{4}$ into $\sin(x)-\cos(x) \quad\Rightarrow\quad y=-\sqrt{2}$

Minimum$\left(\frac{7\pi}{4},-\sqrt{2}\right)$

Maximum$\left(\frac{3\pi}{4},\sqrt{2}\right)$, Minimum$\left(\frac{7\pi}{4},-\sqrt{2}\right)$

Graph

Plotting: $y = \sin(x) - \cos(x), 0 \le x \le 2\pi$